



Performance Investigation of Robust Antenna Array Processor for minimizing Mutual Coupling Effect and Steering Angle Disparity

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ABSTRACT

Antenna array processor's performance is limited extremely in the existence of two array imperfections; mutual coupling (MC) among antenna array elements and steering angle disparity (SAD). That's why; robustness for antenna array processor has come to be a vital issue. The key shortcomings of conventional robust loading techniques are that there is no such algorithm to minimize both MC effect and SAD acting at the same time and no perfect perception is concerned to choose the level of loading according to error level. The proposed robust loading technique maximally minimizes the MC effect and SAD acting alone or in accompany. It is also able to cancel interference from signal of interests and to select loading levels spontaneously according to error level that makes it as a completely automatic loading method. The performance of the presented loading technique is evaluated in the existence of MC effect as well as SAD through numerical examples and fitness among existing robust array processors have also been compared. This paper also shows that the demonstrated method performs superior than the existing conventional method.

1. Introduction

Beamforming has turned into a common word in array signal processing with widespread applications in the field of robotics, biomedical analysis, communications system, cellular networks, geophysical exploration, radar, sonar, acoustics, medical imaging, astronomy, seismology [1-3]. As a result, an effective algorithm is required to estimate and detect desired signals with cancelling interferences and minimizes noise power in any worst situations.

A single antenna has a much lower gain which is not practically implementable. So, there is a normal tendency to develop antenna arrays with a larger number of antenna elements for better performance and resolution. Because of doing this, the antenna elements of array are located closer to each other. When an antenna element is radiating energy, some of the energy in one element is coupled into a neighboring element. This effect is called mutual coupling. In combination with scattering effects from the antenna tower and nearby structures the radiation characteristics of an

antenna can vary significantly from the stand-alone antenna characteristics [4-6]. The effect of the mutual coupling among antenna array elements affects the performance of the array which the performance depends on the types of antenna and its parameter designs, relative positioning of array elements, feeding method of the elements of array and array scan volume [4]. The mutual coupling between antenna elements affects the antenna parameters like terminal impedances, reflection coefficients and hence the antenna array performance in terms of radiation characteristics, output signal-to-interference noise ratio (SINR) and antenna gain [5].

The direction at which the array has maximum power is said to be the look direction. Thus, the array has the maximum gain in that direction. When input signals are collective without any gain and phase change, the look direction is broadside to linear array, that is, perpendicular to the line assembling all elements of the array [5]. Knowledge of the look direction is used to constrain the array response in the signal direction such that the signal arriving from the look direction is passed through the array processor without any distortion but all other signals except look direction are mitigated. The array weights of MVDR beamformer are estimated by minimizing the mean output power subject to the look direction constraint. The processor maximizes the output signal to noise ratio by cancelling all interferences. This is happened by regulating the antenna pattern to cancel these interferences with the main beam pointed in the signal direction. Consequently, the system is said to be employing an optimal antenna when the gain and the phase of the signal induced on each antenna array element are adjusted to achieve the maximum output SINR [5-6]. A direction source is treated as interference and may be attenuated if it is not in the look direction. The amount of attenuation depends on the power of the signal and the amount of error.

Antenna array processors suffer from performance deterioration in the presence of mutual coupling between antenna array sensors and look direction error which is also known as steering angle disparity. It has been found that the performance of an adaptive array antenna is highly affected by these array imperfections, which is particularly serious for small array spacing and large directional error; it decreases the output signal-to-interference-noise ratio. A radiation pattern for a given set of properties, such as main beam direction and gain, position and level of nulls, maximum side lobe level in a specified region are also affected by MC effect and look direction error (LDE) [1-3], [14-18].

The MVDR beamformer is a spatial filter that maximizes the array output signal-to-interference-plus-noise ratio by providing the true covariance matrix and the array steering vector with accurately known value. However, the covariance matrix can be erroneously calculated due to look direction errors and presence of mutual coupling between antenna array elements. Whenever these factors exist, performance degradation for MVDR beamformer has been occurred. This degradation becomes much more serious if the correlation matrix is estimated while the signal-of-interest (SOI) is present in the array output. Therefore, beamforming in smart antenna needs robustness to minimize array imperfections [6-7], [17].

In recent decades many methodologies have been proposed to improve the robustness of the beamforming in the presence of mutual coupling and look direction errors but in a separate manner. These approaches include Fixed Diagonal Loading, Eigen-value based Diagonal Loading, MVDR Diagonal Loading, General Diagonal Loading, New Variable Loading technique to minimize LDE [1-3], [8]. Fixed diagonal loading (FDL) technique is used to minimize the mutual coupling effect [3]. A new method of the mutual impedance calculation is introduced for the compensation of mutual coupling effect in a dipole antenna array has been addressed in [11]. To minimize the MC effect, two approaches towards beamforming based on constrained Particle Swarm Optimization (PSO) are presented in [13] which require more computational cost. In [19], unraveling boundary value problem based compensation matrix method is used to compensate the mutual coupling effect in a uniform linear array leads to an admirable performance of DOA estimation with greater accuracy and resolution. Generalized loading technique of the covariance matrix is used in the presence of random steering vector error [15]. The robust techniques described in [1-2], [4-9] are concerned with look direction error and [3], [11-15] are apprehensive for mutual coupling effect individually. In [20], decomposition of the actual steering vectors based robust capon beamforming is applied where steering vector error is dominated by large direction- of- arrival mismatch which provides superior performance than existing robust capon beamformer. The aim of this paper is to establish an algorithm to minimize steering angle error and mutual coupling effect simultaneously and to achieve better performance than the processor [6].

This paper is structured in the succeeding manner. Section II defines the process of signal formation including array system scheme and array geometrical configuration. Different beamforming techniques are described in section III. Section IV enlightens on our proposed robust antenna array processor. Numerical outcomes of our proposed robust antenna array processor with existing processors in the presence of MC and SAD are shown in section V. In section VI, the conclusion is made.

2. Signal Demonstration

Consider a uniform linear antenna array system consisting with L antenna element which is shown in Fig. 1 where the spacing distance between adjacent array elements is equal to the half wavelength.

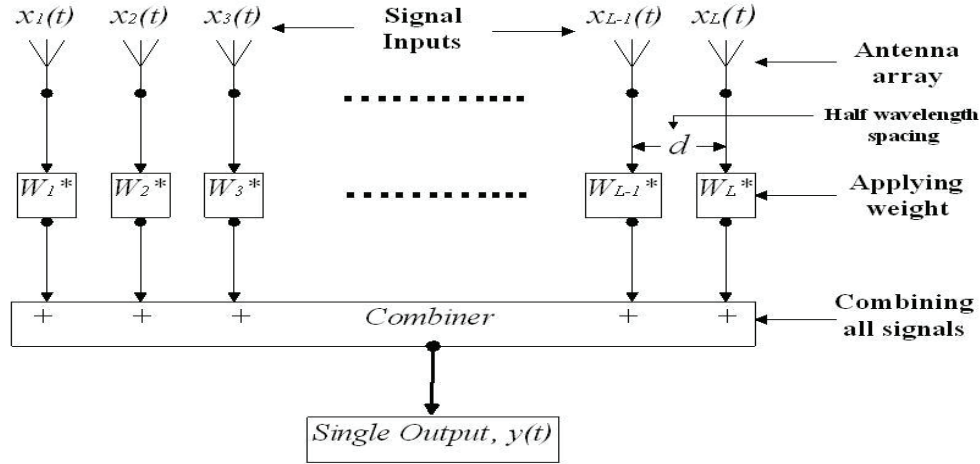


Figure 1: Antenna array system consisting with L antenna elements.

A. Array System Scheme

An expression for the array output is given by [5]

$$y(t) = \sum_{l=1}^L W_l^* x_l(t) \dots \dots \dots (1)$$

Where “ * ” sign represents the complex conjugate. The conjugate of complex weights is used to simplify the mathematical symbolization. Denoting the weights of the array system using vector notation are as

$$W = [W_1, W_2, W_3, \dots, W_L]^T \dots \dots \dots (2)$$

The input signals induced on all elements as

$$x(t) = [x_1(t), x_2(t), x_3(t), \dots, x_L(t)]^T \dots \dots \dots (3)$$

The output of the array system becomes

$$y(t) = W^H x(t) \dots \dots \dots (4)$$

Where, the superscript T and H indicate the transposition and the complex conjugate transposition of a vector or matrix, respectively.

The output power of the array at any time t is given by [5]

$$P(t) = W^H x(t)x^H(t)W \dots \dots \dots (5)$$

The array correlation matrix defined by [5] is given as

$$R = E[x(t)x^H(t)] \dots \dots \dots (6)$$

Where, $E[.]$ represents the expectation operator. Elements of this matrix indicate the correlation between various elements. For example, R_{ij} indicates the correlation between i^{th} and j^{th} element of the antenna array.

B. Array Geometrical Configuration

The array geometry for L element Uniform Linear Array (ULA) is given below. In Fig. 2, we have considered an L element ULA with element spacing d . We have also assumed that the array elements are brought into line with the x -axis such that the first element is positioned at the origin.

Now, the time taken by a plane wave incoming from k^{th} source and measured from l^{th} element is [5]

$$\tau_1 = \frac{d}{c}(L-1)\cos\theta \dots \dots \dots (7)$$

Let S_k represents the steering vector related with the k^{th} source.

For an array with all identical elements, steering vector is defined as

$$S_k = [\exp(j2\pi f_0 \tau_1(\phi_k, \theta_k)), \exp(j2\pi f_0 \tau_2(\phi_k, \theta_k)), \dots, \exp(j2\pi f_0 \tau_L(\phi_k, \theta_k))]^T \dots \dots \dots (8)$$

Note that when the first element of the array is at the origin of the coordinate system, the first element of the steering vector is identical to unity [1].

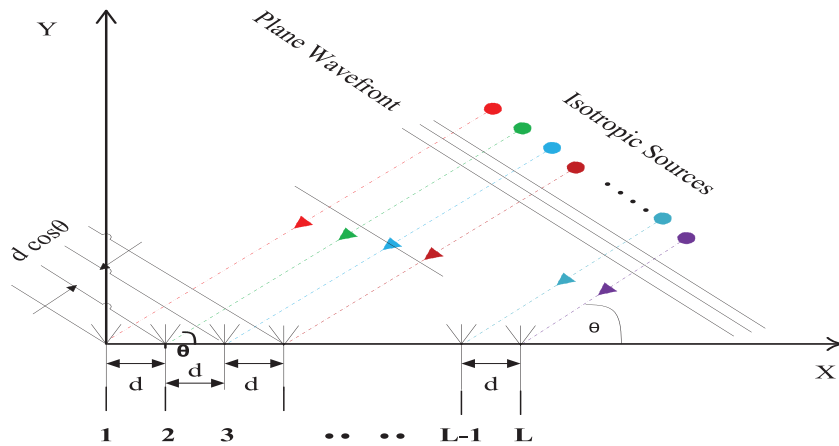


Figure 2: Array geometry for uniform linear array with element spacing d .

3. Beamforming Process

Smart antenna normally practices an array of antenna elements. Each antenna used to form an array is known as array element. Different elements of an array induce different signals and then these signals are needed to be combined to form a single output. This combining process of different signals is recognized as beamforming. It is done in such a way that any predefined radiation pattern can be created. Beamforming may be used at transmitting antenna and receiving antenna to accomplish spatial filtering capability. It is generally categorized into two fundamental categories (I) Conventional Beamforming technique and (II) Minimum Variance Distortionless Response (MVDR) Beamforming technique.

A. Conventional Beamforming Technique

The conventional (switched beam or fixed) beamformer, sometimes also known as the delay-and-sum beamformer, has weights of equal magnitudes. The phases are selected to steer the array in a particular direction (ϕ_0, θ_0) known as look direction to combine the signals from different sensors of different directions. Here, the S_0 denotes the steering vector in the look direction given by [5]

$$S_0 = [\exp(j2\pi f_0 \tau_1(\phi_0, \theta_0)), \exp(j2\pi f_0 \tau_2(\phi_0, \theta_0)), \dots, \exp(j2\pi f_0 \tau_L(\phi_0, \theta_0))]^T \dots \dots \dots (9)$$

The array weights are given by [5]

$$W_C = \frac{1}{L} S_0 \dots \dots \dots (10)$$

The response of a processor in a direction (ϕ, θ) is obtained by using (4) that is, taking the dot product of the weight vector with the steering vector $S(\phi, \theta)$. With the weights given by (10), the output response of the processor, $y(\phi, \theta)$ is given by

$$y(\phi, \theta) = W_C^H S(\phi, \theta) \dots \dots \dots (11)$$

Therefore, the final expression of the output response is become

$$y(\phi, \theta) = \frac{1}{L} S_0^H S(\phi, \theta) \dots \dots \dots (12)$$

B. MVDR Beamforming Technique

The conventional scheme described in the former section requires knowledge of the directions of interference sources, and the beamformer using the weights estimated by this scheme does not maximize the output SINR. The MVDR beamforming method described in this section overcomes this limitation and maximizes the output SINR in the absence of errors and does not require knowledge of directions of interference sources and power levels of interference sources as well as the level of the background noise power to maximize the output SINR. It only requires the desired signal direction of arrival (DOA) [1], [5].

We now discuss an MVDR beamformer with its weights with constraints. To have a unit response in the look direction, let the array weights be constrained, that is,

$$\hat{W}^H S_0 = 1 \dots \dots \dots (13)$$

An expression for the weights of the constrained processor for the case is given by [1]

$$\hat{W} = \frac{R^{-1} S_0}{S_0^H R^{-1} S_0} \dots \dots \dots (14)$$

These weights are the solution of the following optimization problem

$$\begin{array}{ll} \text{Minimize} & W^H R W \\ & W \dots \dots \dots (15) \\ \text{Subject to} & W^H S_0 = 1 \end{array}$$

C. Mutual Coupling Matrix Establishment

In order to evaluate the effect of mutual coupling on the performance of the proposed beamformer, we have incorporated a mutual coupling (MC) matrix in the model for the received signal, revising array correlation matrix to [4]

$$R = E[X_C(t) X_C^H(t)] \dots \dots \dots (16)$$

Where C is a mutual coupling matrix is given by [4].

$$C = [Z_A + Z_L](Z_C + Z_L I_N)^{-1} \dots \dots \dots (17)$$

Where the term Z_A is the sensor impedance without mutual coupling, Z_A is the impedance of the receiver at each sensor which is taken to be 50Ω and I_N is the identity matrix. Now, considering the case of an antenna array with the side-by-side configuration, the mutual impedance matrix Z_C is given by [4]

$$Z_C = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1M} \\ Z_{21} & Z_{22} & \dots & Z_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{M1} & Z_{M2} & \dots & Z_{MM} \end{bmatrix} \dots \dots \dots (18)$$

Where the entry $Z_{M,N} = 1,2,3, \dots, M$ is given by

$$Z_{mn} = \left\{ \begin{array}{l} 30[0.5772 + \ln(2\kappa\gamma) - Ci(2\kappa\gamma)] + \\ j[30(Si(2\kappa\gamma))] = Z_A \text{ for } m=n \\ 30[2Ci(\mu_o) - Ci(\mu_1) - Ci(\mu_2)] - j \\ [30(2Si(\mu_o) - Si(\mu_1) - Si(\mu_2))] \text{ for } m \neq n \end{array} \right\} \dots \dots \dots (19)$$

Here, the terms $k = 2\pi / \lambda$, $\gamma = \lambda / 2$, $\mu_0 = kd$, $\mu_1 = k(\sqrt{d^2 + \gamma^2} + \gamma)$, $\mu_2 = k(\sqrt{d^2 + \gamma^2} - \gamma)$, and d denotes the distance between the two array sensors and $C_i(\alpha) = \int_{-\infty}^{\alpha} (\cos(x)/x)dx$ and $S_i(\alpha) = \int_0^{\alpha} (\sin(x)/x)dx$ are the cosine and sine integrals respectively.

4. Proposed Robust Beamforming Algorithm

Let the steering vector in the actual signal direction be symbolized by S_{ac} . In this the array correction matrix R_{ac} case is given by [1]

$$R_{ac} = p_S S_{oac} S_{oac}^H + p_I S_I S_I^H + \sigma^2_n I \dots \dots \dots (20)$$

An alternative and simple methodology to the fully automatic computation of the diagonal loading level is New Variable Loading (NVL) loading. The conventional sample covariance matrix used in MVDR beamforming is replaced by an enhanced estimate obtained via shrinkage method [1].

The expression of robust correlation matrix is given by

$$R_{new} = R + R^{-1} * \beta * I \dots \dots \dots (21)$$

Where, η and β are the shrinkage parameters [1].

$$\eta = \min\left[v \frac{\rho}{\|R_{f_{ac}} - v * I\|^2}, v\right] \dots \dots \dots (22)$$

And

$$\beta = 1 + \eta \dots \dots \dots (23)$$

Where

$$v = tr(R_{ac}) / L \dots \dots \dots (24)$$

And

$$\rho = \|R_{as} - R_{ac}\|_F \dots \dots \dots (25)$$

Here, R_{as} and R_{ac} are array correlation matrix without error and with error respectively, $tr(.)$ and $\|.\|_F$ represent the trace operator and Frobinius norm of a matrix respectively.

5. Numerical Outcomes

With the intention of describing the mutual coupling effect and steering angle disparity acting alone on beamforming or acting together on beamforming, a linear array of isotropic, identical array elements with equal excitations and a progressive phase difference of eight equi-spaced array elements with half wavelength spacing, unity input signal power, beam pointing direction of 100°, four interferences at a position of 40°, 70°, 130° and 160° with each interferences power of 10.0 Watt and noise label of 0.01 Watt have been considered.

It is found from Fig. 3 that the radiation pattern drops to a lower value of less than -100dB at 20°, 70° and 150° i.e. the direction of interference signals for the MVDR beamformer. There have no significant drops in the radiation pattern of conventional beamformer at the direction of interference sources. It clarifies that the MVDR beamformer gives nulls at interference positions whereas the conventional beamformer doesn't. So, MVDR beamformer is suppressing the interferences without causing any distortion of source signal whereas the conventional beamformer isn't. This proves that the MVDR beamformer has better interferences cancellation capability than the conventional beamformer.

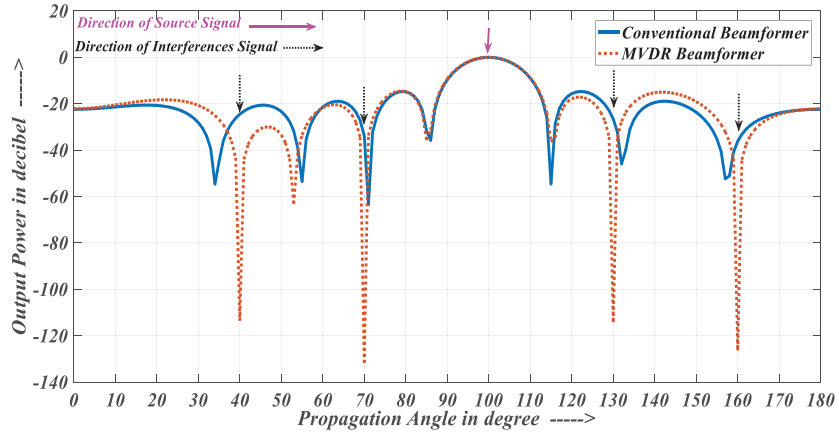


Figure 3: Radiation pattern of conventional beamformer and MVDR beamformer for no. of antenna array elements= 8, the spacing between antenna array elements= half wave length, the power level of signal source= 1.0 W, the direction of signal source= 100°, no. of interference present= 4, the power level of each interference source = 10.0 W, the direction of interference sources= [40°, 70°, 130°, 160°], the power level background noise= 0.01 W

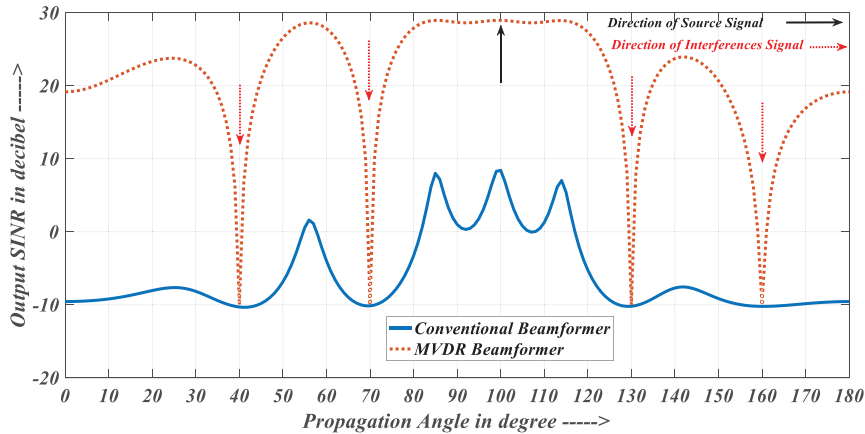


Figure 4: Output SINR pattern of conventional beamformer and MVDR beamformer for the same scenario of Fig. 3 .

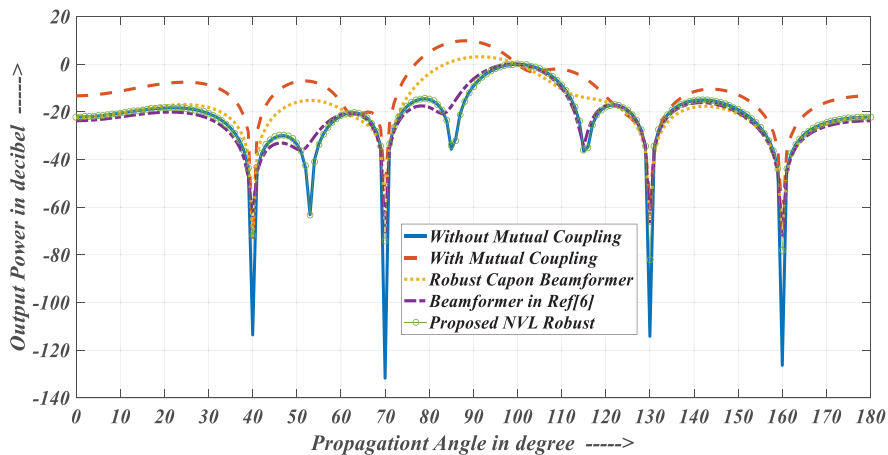


Figure 5: Radiation pattern of MVDR beamformer in the presence of MC for no. of antenna array elements= 8, the spacing between antenna array elements= half wave length, the power level of signal source= 1.0 W, the direction of

signal source= 100° , no. of interference present= 4, the power level of each interference source = 10.0 W, the direction of interference sources= $[40^\circ, 70^\circ, 130^\circ, 160^\circ]$, the power level background noise= 0.01 W

Figure 4 shows the output SINR pattern of conventional beamformer and MVDR beamformer. It is observed that MVDR beamformer gives a flat region of the output SINR with a level of approximately 28dB in the beam pointing direction whereas the conventional beamformer has ripple values in the beam pointing direction with a maximum value of approximately 8dB. It is also observed from the figure that the MVDR beamformer provides minimum output SINR at the direction of all four interference sources with a level of nearly -10dB which proves the optimal performance of the beamforming process. So, it is proved that the MVDR beamformer maximizes the output SINR without affecting the signal whereas the conventional beamformer doesn't.

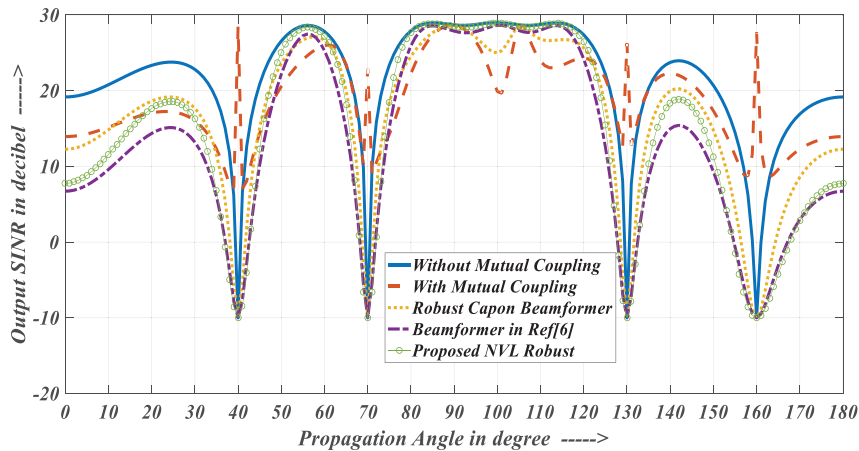


Figure 6: Output SINR pattern of MVDR beamformer in the presence of MC for the same scenario of Fig. 5.

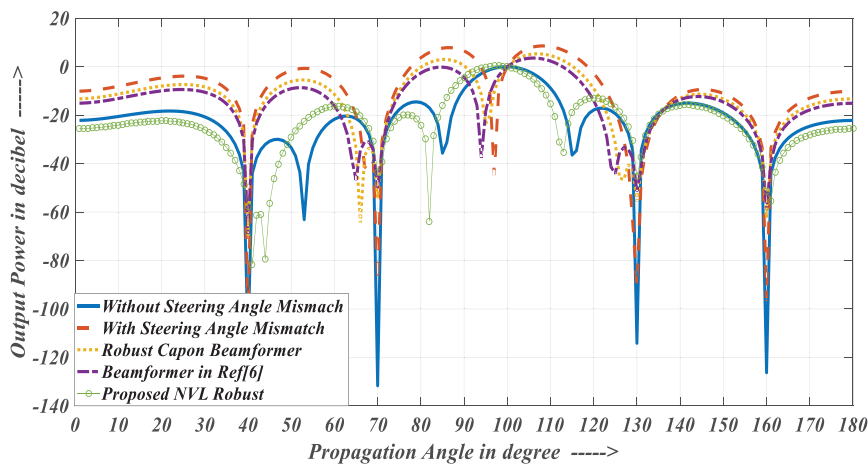


Figure 7: Radiation pattern of MVDR beamformer in the presence of 3° SAD for the same scenario of Fig. 5.

Figure 5 shows the effect of MC on radiation pattern of MVDR beamformer and the robustness of proposed NVL loading technique over other two loading techniques. One found from this figure that the side lobe level has been increased ominously due to the presence of MC. The excessive power loss in the undesired direction due to the increasing side lobe level is very much prohibited for optimal performance of beamformer. One also found that the proposed NVL robust beats MC effect than the other two loading techniques by suppressing the side lobe level and prevents unwanted power loss.

Figure 6 points toward the effect of MC on to the output SINR of MVDR beamformer. It is found from the Fig.7 that the output SINR at beam pointing direction degrades very much whereas the output SINR at the direction of interferences upgrades due to the presence of MC between antenna array elements. It is also found that our proposed NVL robust technique gives higher output SINR in beam pointing direction and lower output SINR in the direction of interferences than the other two loading techniques. This establishes the supremacy of our proposed loading technique over robust capon beamformer and beamformer discussed in [6].

Figure 7 provides information about the effect of SAD on output power of MVDR beamformer and performance comparison in SAD compensation among different loading techniques. We see from the Fig.8 that the main beam shape has been shifted which may cause misjudgment between signal of interest (SOI) and signal of not interest (SONI) and side lobe level has been increased which leads loss of power in undesired direction. The output power pattern found when SAD is not present is being regained by using different loading techniques. It is seen among the three loading techniques that our proposed NVL loading technique is more sensitive in SAD compensation than robust capon beamformer and beamformer discussed in [6].

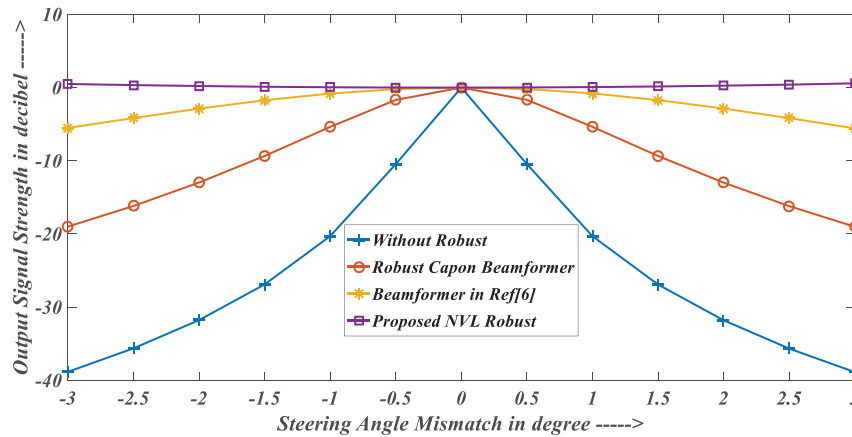


Figure 8: Output signal strength of MVDR beamformer in the presence of SAD for the same scenario of Fig. 5 .

Figure 8 illustrates the SAD effect on output signal strength of MVDR beamformer and compares the performance of different loading techniques in minimizing the SAD effect. It is seen from this figure that output signal strength for 0° SAD is unity as expected for the desired source but decreases with the increasing of SAD in both directions. The output signal strength obtained by applying our proposed NVL loading technique for different degree of SAD provides better performance than the other two loading techniques.

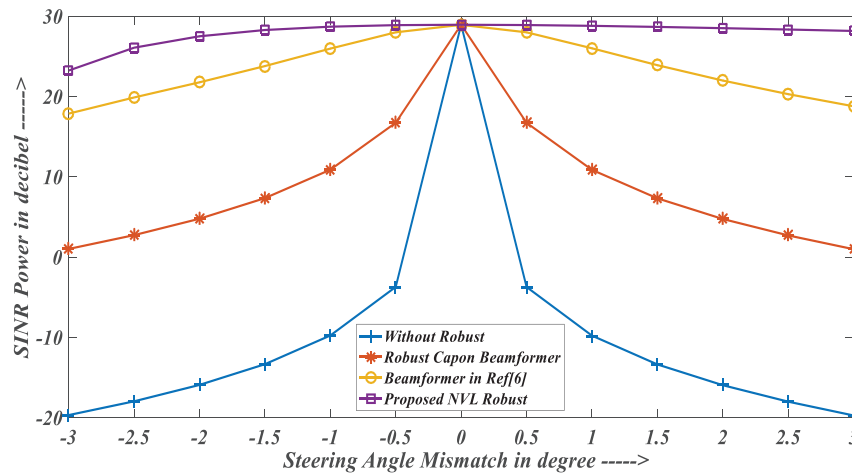


Figure 9: Output SINR power of MVDR beamformer in the presence of SAD for the same scenario of Fig. 5 .

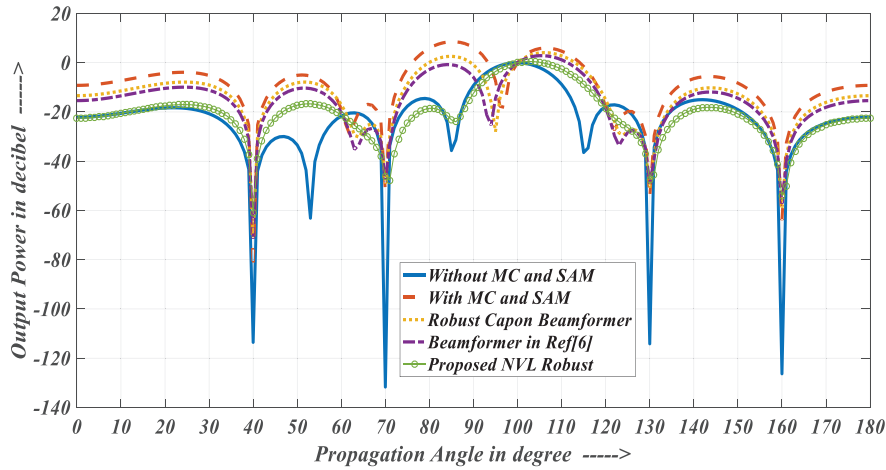


Figure 10: Radiation pattern of MVDR beamformer in the presence of both MC and SAD for the same scenario of Fig. 5.

Figure 9 demonstrates the simulation results in terms of output SINR power of MVDR beamformer using robust capon beamformer, beamformer discussed in [6] and the proposed NVL robust beamformer versus steering angle disparity. It is clearly observed that the output SINR power is highly affected by SAD of higher value. For a value of 3° SAD, the output SINR drops to -20dB approximately whereas robust capon beamformer increases the label of output SINR nearly 2dB and beamformer discussed in [6] increases output SINR around 18dB and our proposed NVL robust loading technique increases output SINR label almost 23dB . So, the simulation results prove that our proposed NVL robust loading technique performs with higher satisfaction of minimization the effect of steering angle disparity on output SINR of MVDR beamformer.

Figure 10 provides the simulation result of output power of MVDR beamformer in the presence of both MC and SAD. The radiation pattern obtained due to the presence of both MC and SAD shows that the main lobe has been shifted as well as side lobe levels have been enlarged significantly. Both of these deviations are not acceptable in beamforming application. It is also observed that the proposed NVL loading technique outperforms the MVDR beamformer in the presence of both array imperfections than the other two loading techniques. Moreover, the simulation results confirm the validity of choosing NVL loading technique to reduce both MC and SAD effect.

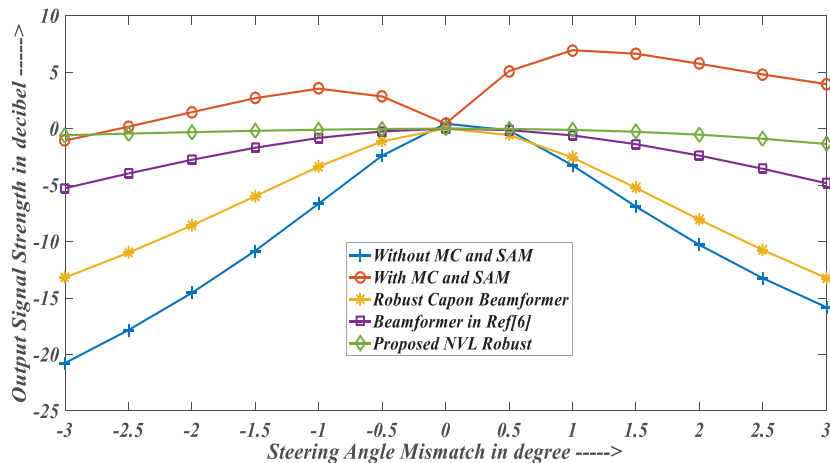


Figure 11: Output signal strength of MVDR beamformer in the presence of both MC and SAD for the same scenario of Fig. 5.

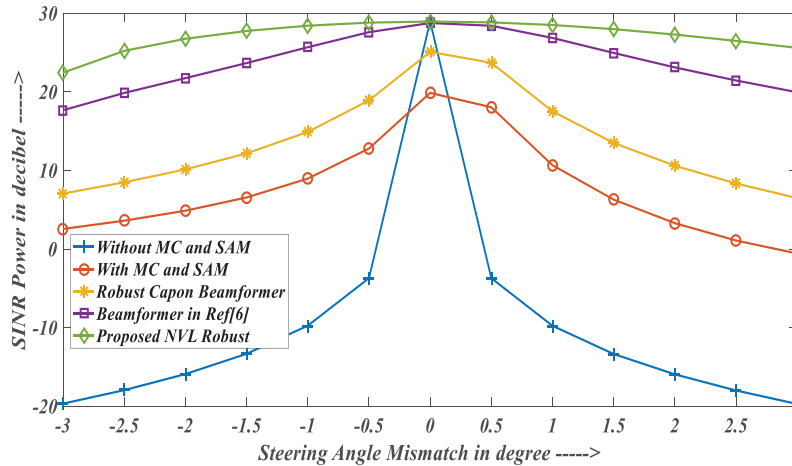


Figure 12: Output SINR power of MVDR beamformer in the presence of both MC and SAD for the same scenario of Fig. 5.

Figure 11 shows the consequence of both MC and SAD on normalized output signal strength of MVDR beamformer. One observes from this figure that the normalized output signal power in the presence of MC as well as SAD is greater than unity. For a constant MC and higher order SAD, the output power is very much affected. The robust capon beamformer and beamformer discussed in [6] minimize these both array imperfections but can't give a constant power for higher error label. In the case of the proposed NVL robust, the normalized output signal power is maintained almost constant for a given MC and different degree of SAD which undoubtedly proves the better robustness eligibility than the other two beamformers.

Figure 12 displays the result of performance comparison of the output SINR power of MVDR beamformer for the robust capon beamformer, beamformer discussed in [6] and proposed NVL robust beamformer in the presence of both MC and SAD. We see from the figure that output SINR decreases as the SAD increases with constant MC. To lessen the effect of both MC and SAD, the proposed NVL robust beamformer gives higher output SINR of the order of 20 to 30dB whatever the error is, resulting in optimal performance of array. However, in the other two beamforming techniques the output SINR drops in a higher value as the SAD increases. So, it is clear that the MVDR beamformer is optimized to maximize the output SINR subject to a constant MC and higher SAD which is decently obtained by using NVL robust.

6. Conclusion

In this paper we have shown the superiority of proposed NVL robust MVDR antenna array processor against mutual coupling effect and steering angle disparity. It can minimize both of these effects in a same algorithm and doesn't need any fixed loading level rather it is a completely automatic loading technique. A number of MATLAB simulations explain the excellent performance of this proposed algorithm. Examples have been presented to compare the performance of NVL with the existing GLC and FDL techniques. It has been shown using computer simulation that the proposed robust processor achieved 6dB higher SINR than the existing processor [6]. We have demonstrated that NVL is very useful in the case where the users of adaptive arrays are most interested for higher gain and better performance.

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